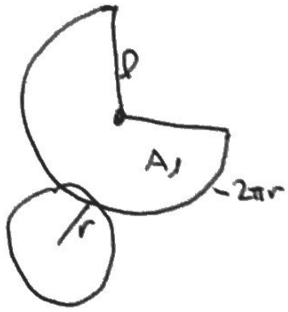
1 a)



b) $\frac{2\pi r}{2\pi \ell} = \frac{A_\ell}{\pi \ell^2}$ by proportions of sectors.

So, $A_\ell = \frac{r}{\ell} \cdot \pi \ell^2 = \pi r \ell$.

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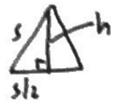
2 a)

Each such hexagon is made of 6 equilateral triangles of side lengths s.



$\left(\frac{s}{2}\right)^2 + h^2 = s^2$, so $h^2 = \frac{3}{4}s^2$, so $h = \frac{\sqrt{3}}{2}s$

$A_\Delta = \frac{1}{2} \cdot \frac{\sqrt{3}}{2}s \cdot s = \frac{\sqrt{3}}{4}s^2$. So the total area of the hexagon is $6A_\Delta = 6\frac{\sqrt{3}}{4}s^2 = \boxed{\frac{3\sqrt{3}}{2}s^2}$

b) The total area of the field is $240 \cdot 360 = 86{,}400 \text{ ft}^2$.

The area of one hexagon is $\frac{3\sqrt{3}}{2} \cdot (7.5)^2 \approx 146.14 \text{ ft}^2$. Since regular hexagons can tessellate the plane, it required approximately $\frac{86{,}400}{146.14} = \boxed{591 \text{ hexagons}}$ to cover the field.

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3 a) We want to minimize surface area, so stack the eight cubes into a 2 cube x 2 cube x 2 cube larger cube.

b) Here we want to maximize surface area, so stack them in a 1 cube x 1 cube x 8 cube column.

a) b)

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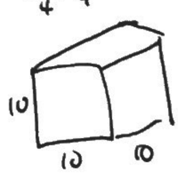
4 a)

$SA_1 = 2(4\cdot4) + 2(4\cdot4) + 2(4\cdot4)$
$= 96 \text{ in}^2$

$V_1 = 4\cdot4\cdot4 = 64 \text{ in}^3$



$SA_2 = 2(10\cdot10) + 2(10\cdot10) + 2(10\cdot10)$
$= 600 \text{ in}^2$

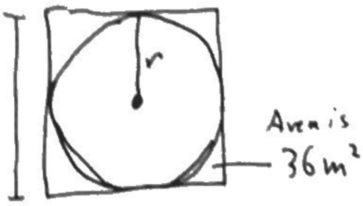
$V_2 = 10\cdot10\cdot10 = 1000 \text{ in}^3$.

$\frac{SA_1}{SA_2} = \frac{96}{600} = \frac{4}{25} = \boxed{\left(\frac{2}{5}\right)^2 = \left(\frac{4}{10}\right)^2}$ $\frac{V_1}{V_2} = \frac{64}{1000} = \frac{8}{125} = \boxed{\left(\frac{2}{5}\right)^3 = \left(\frac{4}{10}\right)^3}$

b) algebraically:

$\frac{SA_2}{SA_1} = \frac{6(2x)^2}{6(x)^2} = \boxed{4}$ $\frac{V_2}{V_1} = \frac{(2x)^3}{(x)^3} = \boxed{8}$

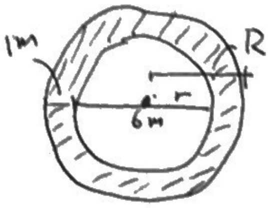
5) Suppose that the fence is fixed in the shape of a square and we want the largest possible garden.



Then $s^2 = 36$, so $s = 6m$. $r = \frac{1}{2}s = 3m$, so

$A_{garden} = \pi r^2 = \boxed{9\pi\ m^2}$

6)



$A_{shaded} = A_{large\,O} - A_{small\,O}$

$= \cancel{\pi R^2}$

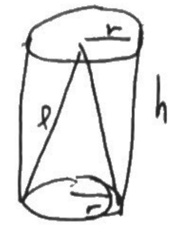
$= \pi R^2 - \pi r^2$

$= \pi(4)^2 - \pi(3)^2$

$= 16\pi - 9\pi$

$= \boxed{7\pi\ m^2}$

7)



We want to show that

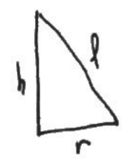
$SA_{cyl} > SA_{cone}$, i.e.

$2\pi r^2 + 2\pi rh > \pi r^2 + \pi rl$, i.e.

$\pi r^2 + 2\pi rh > \pi rl$, i.e.

$r + 2h > l$.

Taking a cross-section, we get the right triangle



, so $h + r > l$ (any 2 sides of a Δ sum to more than the third). Since $h > 0$, this tells us that $2h + r > h + r > l$.

9) See activity in class & HW.

8) a) False

b) Area of medium pizza of radius r:

$\pi r^2 = \frac{1}{3}\pi\left(\frac{x}{2}\right)^2$ ← $\frac{1}{3}$(area of large pizza)

$r^2 = \frac{1}{12}x^2$

$\boxed{r = \frac{1}{2\sqrt{3}}x}$

10) a) False, the base is not a polygon. Prisms are examples of polyhedra.

b) True.

c) False. For example, the lateral sides of a triangular prism have no parallel face.

d) True. There are 3 edges in each base and 3 edges connecting the bases.

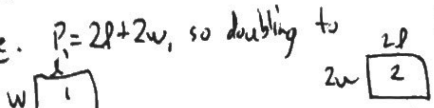
e) True. There is 1 rectangular side for each edge of the base of a prism.

f) True. The base and the 3 lateral faces are all triangular.

g) True. These pyramids have 1 lateral face for each ~~bas~~ edge in the base and 1 base.

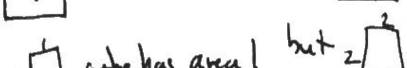
h) False. 1 face per edge of the base and 2 bases is $n+2$ faces.

i) True. $P_1 = 2l + 2w$, so doubling to



has $P_2 = 4l + 4W = 2P_1$.

j) False.



~~cube~~ square has area 1 but square has area 4.

k) False. This quadruples the SA.

l) False. This multiplies the volume by a factor of 8.

11) **Neither.** They don't have two congruent bases, so they aren't ~~cones~~ prisms, but also don't have triangular lateral ~~sides~~ faces, ~~they~~ so aren't pyramids.

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12) $24 \text{ fl oz} \cdot \frac{1 \text{ c}}{8 \text{ fl oz}} \cdot \frac{1 \text{ gal}}{16 \text{ c}} \cdot \frac{3.79 \text{ L}}{1 \text{ gal}} \cdot \frac{60}{100} \text{ full} = \boxed{0.426 \text{ L}}$

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13) 1 gallon = 4 quarts = 8 pints = 16 cups = 128 oz. These units measure volume.

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14) Liters are not a measure of mass. To measure the mass you either need to know the density of water to convert volume to mass or you need to use a scale of some kind to get a measurement in kg.

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15) a) $300{,}000 \frac{\text{km}}{\text{s}} \cdot \frac{0.62 \text{ mi}}{1 \text{ km}} \cdot \frac{60 \text{ s}}{1 \text{ min}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \cdot \frac{24 \text{ hr}}{1 \text{ day}} \cdot \frac{365 \text{ day}}{1 \text{ yr}} = 5.86 \times 10^{12}$ miles/year,

so 1 lightyear is $\boxed{5.86 \times 10^{12} \text{ miles}}$

b) $4.22 \text{ ly} \cdot 5.86 \times 10^{12} \text{ mi/ly} = \boxed{2.47 \times 10^{13} \text{ miles}}$